# Table of Contents

1. Introduction

2. People

3. Publications, and Conference and Lecture Presentations

4. Research: Development Reduced Order Models (ROMs) for Wind Turbine / Plants: Reduced-Order Characterization of Aerodynamic-Structural Interaction
   4.1. The Common ODE Framework (CODEF) Multiphysics Model for Wind Turbine/Wind Farm Dynamics, and its Modules
   4.2. The Generalized Timoshenko Beam Model (GTBM)
   4.3. The DRD-BEM model
   4.4. Aeroelastic Reduced-Order Characterization - Objectives and Methodology
   4.5. Characterization of Pulses Representing Typical Atmospheric Flow Oscillations
   4.6. Energy Transfer Characterization of the pulse (Stable Oscillatory Regime)
   4.7. Energy Transfer Characterization of the pulse (Unstable Oscillatory Regime)

5. Funds Usage

6. Concluding Remarks

7. References
1. Introduction

During this reporting period, the Richard and Elizabeth Henes Endowed Associated Professorship in Wind Energy provided graduate student support and equipment needs for the continued development of research in the area of advanced simulation and analysis of the aeroelastic dynamics of wind turbine rotors, and in particular, their interaction with other turbines within the wind farm collective. The ultimate technical goal is to develop representative reduced order models (ROM’s) that capture the critical coupled dynamics of the aerodynamic-mechanical-electrical systems, and create an advanced nonlinear control and agents/informatics architecture that will seamlessly integrate and harmonize Energy Storage Systems (ESS). The nonlinear control design will encompass a distributed/decentralized approach that identifies where and how much ESS is required to operate efficiently. We have leveraged on our wind turbine/farm codes R&D program at Michigan Tech University to develop a novel multiphysics computationally-efficient model: CODEF that can capture the interaction of wind plant atmospheric inflow with wind turbine wakes, but at a computational cost much lower than the current high-fidelity unsteady RANS and LES models.

A fundamental component of this project is the use of CODEF to obtain a Reduced-Order Characterization of the aero-elasto-inertial dynamics of the rotor oscillatory behavior when stimulated by pulses in the wind of controlled amplitude and time span. Through this experiments, we gained a better understanding of the underlying physics of the principle of energy transfer between the kinetic energy of the wind speed pulse and the elasto-inertial energy accumulated in the rotor. These experiments, will provide extremely valuable information to the wind power control community to produce ROMs that could be solved in real time to develop predictive control strategies. The new RO characterization is capable of representing all the essential modes of the rotor’s dynamics via a compact and fast algorithm, giving the MEEM Department at Michigan Tech a recognizable edge over other research institutions in the field.
2. People

**Apurva Baruah**

PhD Student - Graduate Research Assistant.

Work on the vortex dynamics and turbine wake simulation aspects of the project.

**Alayna Farrell**

PhD Student - Graduate Research Assistant.

Work on the fluid-structure interaction dynamics aspects of the project, and atmospheric turbulence induced effects.

**Dr. Sarah Jalal**

PhD Graduate.

Work on the aeroelastic and blade vibrational dynamics aspects of the project.
3. Publications, Conference and Lecture Presentations

Journal, Peer Reviewed Proceeding Papers, Thesis, and Technical Reports:


Conference and Lecture Presentations:

4. Research: Development Reduced Order Models (ROMs) for Wind Turbine / Plants: Reduced-Order Characterization of Aerodynamic-Structural Interaction

4.1. The CODEF Multiphysics Model for Wind Turbine/Wind Farm Dynamics, and its Modules

The Common ODE Framework (CODEF) is an Adaptive Dynamic Multiphysics modeling technique for wind turbine dynamics via a Multivariable ODE Solution in time. Based on using non-linear adaptive variable-timestep /variable-order algorithms to solve a master ODE system, CODEF gathers together the equations associated with the different modules modeling rotor flow, blade structure, control system, and electromechanical devices (see Fig. 1-4.1). By monitoring the local truncation error at every timestep, CODEF provides a natural way of integrating simultaneously all aspects of the multiphysics problem, improving the efficiency and ensuring the stability of the time-marching scheme. These unique features make CODEF capable of modeling the aeroelastic dynamics associated with active control actions. Moreover, CODEF ODE system could be expanded to interphase with the equations simulating the electrical interaction within the intra-farm microgrid, the wind-farm collective control system, and the farm-scale collective flow through the Gaussian-Core Vortex-Lattice Model (GVLM) explained below.

Fig. 1-4.1: Block schematics of CODEF, including expansion to wind farm simulation DRD-BE-GVLM.

GTBM: Generalized Timoshenko Beam Model
DRD-BE-GVLM: Dynamic Rotor Deformation - Blade Element - Gaussian Vortex Lattice Model
Detailed description of CODEF key modules:

1. Moderate-Order High Fidelity Blade Structural Model: The GTBM technique allows for a reduction of the 3D structural problem through a set of 2-D linear pre-solutions on blade sections along the span into a 1-D non-linear time-dependent solution on an equivalent beam written as an ODE system. A brief description of the GTBM technique is provided in section 4.2 here, full details could be found in Ponta, et al. [1], and the references within.

2. Moderate-Order High Fidelity Rotor Flow Model: DRD-BE technique provides a fully-coupled aero-elastic model based on a complete reformulation of the Blade-Element technique including rotor deformation. In its initial form, the DRD-BEM, was combined with a classic Momentum Streamtube model for an individual turbine (see Ponta, et al. [1]). It could also interphase with the farm-scale collective flow through integration with the GVLM technique explained below. A brief description of the DRD-BEM is provided in section 4.3 here, full details could be found in Ponta, et al. [1]

3. Moderate-Order High-Fidelity Collective Farm-Flow Model: The Gaussian-Core Vortex-Lattice Model (GVLM) is a high-fidelity alternative for farm collective flow-field simulation, with moderate computational cost. GVLM consists of an ensemble of the individual Gaussian-core vortex filaments on the lattices generated by all turbines in the farm. The ensemble is shared by the collective Farm-Flow GVLM and by all the DRD-BE-GVLMs of the individual turbines. Gaussian distribution of vorticity in the vortex core accounts for the natural viscous decay of vortex filaments, freeing the memory from vortices that have already dissipated. This substantially reduces the computational cost versus classic vortex models based on the singularity concentration of vorticity in vortex filaments that never decay. It also provides a much more realistic representation of the interaction of vortices in close proximity, by avoiding the unrealistic, extremely high values of vortex-filament induced velocity given by the Biot-Savart law. Detailed descriptions of the Vortex-Lattice technique, and the Gaussian-Core Vortex theory could be found in Ponta and Jacovkis [2], and Ponta [3], among other publications.

4. Modular Multiphysics Interface: CODEF is Integration-Ready with ODE modules modeling the dynamics of the electro-mechanical devices and the control system at individual-turbine scale, as well as, expansions to include ODE equations modeling the electrical interaction within the intra-farm microgrid, and the wind-farm collective control system.
4.2 The Generalized Timoshenko Beam Model (GTBM)

GTBM uses the same variables of classical Timoshenko beam theory, but the hypothesis of beam sections remaining planar is abandoned. Instead, the real warping of the deformed section is interpolated by a 2-D finite-element mesh; the strain energy is rewritten in terms of the classical 1-D variables, and pre-solved.

Example of a typical blade design used in current commercial turbines.

Example of finite-element 2-D meshing of the internal structure of one of the blade sections used on the dimensional-reduction computations.

The dimensional reduction produces a fully populated 6x6 stiffness matrix for an equivalent beam, which includes all coupled-deformation modes like the bending-twist mode. It allows accurate modeling of the blade structure as a 1-D finite-element problem to be solved at each time-step of the ODE solution at a reduced computational cost.
4.3 The DRD-BEM model

In the (Dynamic Rotor Deformation) DRD-BEM wind velocity is transformed through a set of orthogonal-matrix linear operators into the system of coordinates of the blade section in its instantaneous deformed configuration. This fully includes the effects of the misalignment induced by large rotations of the airfoil sections on the computation of aerodynamic forces on the blade element. This technique includes misalignments by deformation, the action of mechanisms like yaw, pitch, main-shaft rotation, changes in wind direction, and design features like blade pre-conforming processes, rotor tilt or coning.

An example of orthogonal-matrix linear operator to account for pitch-control actuation:

\[
C_{\theta_p} = \begin{bmatrix}
\cos(-\theta_p) & \sin(-\theta_p) & 0 \\
-\sin(-\theta_p) & \cos(-\theta_p) & 0 \\
0 & 0 & 1
\end{bmatrix}
\]
Once the wind velocity is modified to account for interference, by application of the velocity-induction coefficients \( a \) and \( a' \), and transformed into the blade element coordinates, velocity components due to structural vibration and the action of mechanisms are added. Aerodynamic forces on blade elements are computed and transformed back to the stream-tube coordinates to compute the interference.

\[
W_{\infty h} = \left( C_{\theta zs} C_{\theta st} C_{\Delta \theta yse} W_{\infty \text{wind}} \right)
\]

\[
W_h = \begin{bmatrix}
W_{\infty h x} (1 - a) \\
W_{\infty h y} + \Omega r_h a' \\
W_{\infty h z}
\end{bmatrix}
\]

\[
W_l = \left( C_{L L} C_{L b} C_{\theta p} C_{\theta en} W_h \right) + \mathbf{v}_{\text{str}} + \mathbf{v}_{\text{mech}}
\]

\[
\delta \mathbf{F}_h = C_{\theta en}^T C_{\theta p}^T C_{L b}^T C_{L L}^T C_{L b a} d \mathbf{F}_{rel} \delta l
\]
4.4 Aeroelastic Reduced-Order Characterization - Objectives and Methodology

We conducted an extensive series of experiments on the aero-elasto-inertial response of wind turbine rotors, using Delft University’s stall-control rotor (see Jaimes [4]) alternative for the NREL-5MW-RWT benchmark turbine. The DU-5MW stall RWT has a nominal wind speed of 15.3 m/s, and a nominal rotational speed of 10.7 rpm. By studying the time evolution of the rotor oscillatory behavior when stimulated by pulses in the wind of controlled amplitude and time span (see example on Fig. 1-4.4), we gained a better understanding of the underlying physics, and obtained a reduced-order characterization of the rotor as an oscillatory system based on the principle of energy transfer between the kinetic energy of the wind speed pulse and the elasto-inertial energy accumulated in the rotor. These experiments, reported in Jalal, Ponta and Baruah [5], and Jalal [6], and Jalal, Ponta, Baruah, and Rajan [7] will provide extremely valuable information to the wind power control community to produce Reduced Order Models (ROMs) that could be solved in real time to develop predictive control strategies.

![Fig. 1-4.4: An example of a wind-speed pulse stimulus of controlled amplitude and time span.](image)

Wind Speed Stability Threshold, and the Role of Aerodynamic Damping

For wind speeds between 5 m/s – 16.5 m/s, we found that the dynamic oscillatory response of the rotor remains stable. The kinetic energy of the wind pulse initiates oscillations and those oscillations become damped by the system, creating a decay oscillatory signal (which is observable in the time evolution of axial component of the blade deflection depicted in Fig. 2-4.4 and Fig. 3-4.4).

Above 17 m/s the rotor, as an oscillatory system, is no longer damping the oscillations but amplifying them, feeding on the energy from the wind flow. After the pulse has passed, the
oscillations continue expanding exponentially until a second damping mechanism starts to act, slowing the growth and reaching an equilibrium where the oscillation amplitude becomes stable and self-sustained.

As we are going to see in detail next, this changing of behavior from stable to unstable oscillatory regimes is associated with a change in sign of the aerodynamic damping. Which depends exclusively on the wind speed mean value.

![Graphs showing time evolution of blade deflection at different wind speeds](image)

**Fig. 2-4.4:** Examples of the time evolution of the axial component of the blade deflection at wind speeds below and above the wind-speed stability threshold.

Between the stable and unstable regimes there is a narrow range of wind speeds that exhibits the classical signs of a transitional behavior, where attenuation and amplification may occur. Here we see two examples (even though several variations are possible).
At the beginning of the transitional regime, the system starts damping the oscillations but they are not completely dissipated, and some residual "band" of oscillation remains. Further on in the regime, expansion occurs followed by a brief stabilization, and then oscillation decay. The same manifestation of sustained residual oscillations is present in the long run.

This regime is characterized by a very rich combination of frequencies whose amplitudes evolve in time in a very complex manner.

![Graphs](image)

**Fig. 3-4.4:** Examples of the time evolution of the axial component of the blade deflection at wind speeds in the narrow range of the transitional regime.
4.5 Characterization of Pulses Representing Typical Atmospheric Flow Oscillations

Our objective was to identify and quantify pulses with a characteristic time span and amplitude that represent typical oscillations in wind speed in a statistical sense. We collected samples of anemometry data from different sources, and classified them in an amplitude versus characteristic time span chart (see Fig. 1-4.5). We found three distinguishable regions in this chart, which are indicated by encircling elliptical dashed lines, namely:

1. Short Pulses (associated to wind flow turbulence): Characterized by pulses which are short enough that occur before the first peak in the blade oscillation occurs. After a very short initial time when the blades displace accumulating energy by their own inertia, they deflect to a maximum, and the kinetic energy content of the wind pulse is accumulated as blade elastic deformation. Energy transfer and subsequent evolution can be characterized only by measuring the instantaneous blade deflection.

2. Pulse-Duration Transitional Zone: Characterized by pulses which are long enough that energy dissipation by aerodynamic damping occurs during the duration of the pulse itself (i.e., only part of the pulse energy goes into elastic energy). Energy transfer and evolution can no longer be characterized only by measuring blade deflection.

3. Long Pulses: Characterized by pulses which are long enough that they act as gradual variations in the flow kinetic energy that are absorbed by the rotor, inducing very small (or even negligible) oscillations.

Fig. 1-4.5: Amplitude versus characteristic time span chart for typical oscillations in wind speed.
4.6 Energy Transfer Characterization of the pulse (Stable Oscillatory Regime)

The axial displacement of the blade $U_{\text{nx}}$, depicted in the upper panel on Fig. 1-4.6 for a wind speed regime of 8m/s, reflects the main component of the instantaneous amount of elastic energy stored in the rotor as an oscillatory system. We used it as an observable to construct a reduced-order characterization of the complex non-linear oscillatory behaviors, in terms of only one degree of freedom. We conducted this experiment for a wide variation of pulses in the Short-Duration zone in the classification chart shown in Fig. 1-4.5, combining different values of pulse amplitude and pulse time span. Table 1-4.6 shows the code of colors and markers used to identify the results for pulses combining different values of amplitude and time span.

Fig. 1-4.6: Upper panel: Time evolution of the axial displacement of the blade at a wind speed regime of 8m/s for pulses combining different values of amplitude and time span. Lower panel: the same curves normalized by the kinetic energy content of their respective pulses.
Table 1-4.6: Code of colors and markers used to identify the results for pulses combining different values of amplitude and time span.

<table>
<thead>
<tr>
<th>ΔW_e [m/s]</th>
<th>1m/s</th>
<th>0.75m/s</th>
<th>0.5m/s</th>
<th>0.25m/s</th>
<th>0.1m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δt_{puls} [s]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>▽ 0.1s</td>
<td>● 0.15s</td>
<td>△ 0.2s</td>
<td>○ 0.25s</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig. 2-4.6: Upper panel: Time evolution of the axial displacement of the blade at a wind speed regime of 12 m/s for pulses combining different values of amplitude and time span. Lower panel: the same curves normalized by the kinetic energy content of their respective pulses.
When normalized by the kinetic energy content of the pulse, as shown in the lower panel on Fig. 1-4.6, all the displacement curves collapse into a unique signal, which scales as the time evolution of the rotor's elastic vibrational energy. This has the units of a flexibility, kN\(^{-1}\), or the inverse of a stiffness \(1/K_{el}\).

**Fig. 3-4.6:** Upper panel: Time evolution of the axial displacement of the blade at a wind speed regime of 16 m/s for pulses combining different values of amplitude and time span. Lower panel: the same curves normalized by the kinetic energy content of their respective pulses.
We repeated the same experiment for several wind speed conditions covering the entire stable regime, finding the same systematic behavior. For example, Fig. 2-4.6 shows the results of the kinetic-energy normalization for a wind speed of 12 m/s, and Fig. 3-4.6 for a wind speed of 16 m/s. The value of that equivalent stiffness mentioned above, $K_{eq}$, which serves as a scaling factor for the rotor's elastic vibrational energy, depends exclusively on the mean wind speed. This is consistent with the average deflection of the blades around which the oscillations occur. The higher the mean wind speed, the higher is the value of this “pre-stiffening”.

**Frequency content in the stable oscillatory regime**

In analyzing the frequency content in the stable regime, we found the appearance of five frequencies (only three of them dominant), which show a different proportional contribution in four different subranges along the stable wind speed range.

**Fig. 4-4.6** shows the spectra for the first subrange, which extends from 5 m/s to 8 m/s, where only the first two frequencies (0.87 Hz and 1.2 Hz) are dominant. The upper panel shows the frequency content all along the time evolution (i.e., until oscillations are totally dissipated), where the 0.87 Hz frequency dominates, with a relatively small contribution of the 1.2 Hz frequency. The lower panel shows the spectrum for the long term part of the same time signals. The 0.87 Hz frequency rapidly decay, while the 1.2 Hz component, even though smaller in the beginning, persists longer. In the long term, there is also a minimal contribution of the non-dominant 4.2 Hz frequency.

**Fig. 5-4.6** shows the spectra for the second subrange, which extends from 9 m/s to 10 m/s. In this subrange the 0.87 Hz frequency is dominant in the early stages, but the contribution of the 1.2 Hz frequency becomes more prominent. The long term spectra for the same signals only show the 1.2 Hz component, persisting even longer than in the first subrange. Again, there is also a minimal contribution of the non-dominant 4.2 Hz frequency.

**Fig. 6-4.6** shows the spectra for the third subrange, which extends from 10.5 m/s to 12.5 m/s. This subrange shows a much richer combination of the five frequencies observed, with three of them dominant. Even though the 4.2 Hz and 5.14 Hz components take long to decay, they are still not dominant. As before, the 0.87 Hz frequency is the most prominent, but the relative contribution of the 1.2 Hz component increased even more compared with the previous subranges. The most remarkable feature of this subrange is the emergence of a third dominant frequency of 2.4 Hz, which is particularly intense at 11 m/s. The 2.4 Hz frequency was only observed as a dominant component in this subrange specifically.
Fig. 7-4.6 shows the spectra for the fourth and last subrange, which extends from 13 m/s to 16 m/s, the contribution is again mostly centered around the 0.87 Hz frequency with a stronger presence of the 1.2 Hz component. An important distinctive feature is the presence of the 0.87 Hz frequency in the long term spectra, which indicates a longer persistence (thus, a lower exponential decay) for this component. This increasingly lower exponential decay of the 0.87 Hz component leads to the end of the stable regime, when the exponential decay approaches zero and then becomes negative. Thus, the aerodynamic damping turns into exponential expansion. In this subrange, the contribution of the other three frequencies becomes minimal.

Fig. 4-4.6: Frequency spectra for the first subrange along the stable regime. Upper panel: complete time evolution. Lower panel: long-term evolution.
Fig. 5-4.6: Frequency spectra for the second subrange along the stable regime. Upper panel: complete time evolution. Lower panel: long-term evolution.

Fig. 6-4.6: Frequency spectra for the third subrange along the stable regime. Upper panel: complete time evolution. Lower panel: long-term evolution.
Semi-log plots of the oscillating signals during the stable regime

Analyzing the semi-log plots of the oscillating signals it becomes clear that their time evolution is characterized by an exponential enveloping curve, manifested as a linear relation when plotted in semi-log axes. **Fig. 8-4.6** shows three examples of semi-log plots of the time evolution of the normalized blade deflection, at selected wind speeds in different subranges of the stable regime. In all the panels, the short term evolution, when the 0.87 Hz frequency dominates, shows a higher exponential decay, $\lambda_1$, given by the slope of the red line. While the lower exponential decay associated with the 1.2 Hz frequency, $\lambda_2$, becomes dominant in the long term. In the middle panel we could see the contribution of the 2.4 Hz frequency overlapping with the 0.87 Hz and the 1.2 Hz components. By applying a bandpass filter to each one of the dominant components the values of their corresponding exponential decays can be measured.
**Fig. 8-4.6:** Semi-log plots of the time evolution of the normalized blade deflection, at selected wind speeds in different subranges of the stable regime.

**Exponential decay of aerodynamic damping in the stable oscillatory regime**

**Fig. 9-4.6** shows the variation of the exponential decay of the aerodynamic damping for the complete range of wind speeds in the stable oscillatory regime. The exponential decay associated to 0.87 Hz frequency ($\lambda_1$) shows a clear connection with the four different subranges associated with the frequency content that we have seen before.
The 1.2 Hz frequency shows an exponential decay which is much lower. Even though it has a lower amplitude in the mix of frequencies, it persists for a longer time, and that is the reason why it is more prominent in the long term spectra.

The 2.4 Hz frequency only appears in the third subrange and it shows a sudden decrease on its exponential decay located at 11 m/s which makes that component persistent for a longer time. It returns to a higher dissipation rate for at wind speeds of 12 m/s.

![Variation of the exponential decay of the aerodynamic damping for the complete range of wind speeds in the stable oscillatory regime.](image)

**Fig. 9-4.6:** Variation of the exponential decay of the aerodynamic damping for the complete range of wind speeds in the stable oscillatory regime.

### 4.7 Energy Transfer Characterization of the pulse (Unstable Oscillatory Regime)

In the unstable regime, $\lambda_1$ had turned negative (that is, the attenuation of the aerodynamic damping had turned into amplification of the oscillatory amplitude), producing an initial exponential expansion. As in the stable regime, the axial displacement of the blade $U_{h_{xx}}$ (depicted in the upper panel on **Fig. 1-4.7** for a wind speed regime of 18 m/s), reflects the main component of the instantaneous amount of elastic energy stored in the rotor as an oscillatory system. The time evolution of the blade deflection shows the same qualitative behavior for all the wind
speeds in this regime (17.5 m/s to 25 m/s), but with different values of the initial exponential expansion. As in the stable regime, \( \lambda_1 \) depends exclusively on the mean wind speed.

![Graph](image)

**Fig. 1-4.7:** Upper panel: Time evolution of the axial displacement of the blade at a wind speed regime of 18 m/s for pulses combining different values of amplitude and time span. Lower panel: the same curves, but delayed in time for the exponential expansion to build up the same amount of energy contained in their respective pulses.

The effect of the kinetic energy content of different pulses manifests as a delay in time, and all the curves for a certain wind speed could be collapsed into one, as it is shown in the lower panel of **Fig. 1-4.7**. That is, the pulse energy provides an initial threshold in the oscillation curve, and from then on, the oscillations continue feeding upon the energy of the mean flow. The delay time could be directly associated with the time required for the exponential expansion to build up the same amount of energy contained in their respective pulses, by extracting it from the mean flow.
Semi-log plots of the oscillating signals during the unstable regime

Fig. 2-4.7 shows semi-log plots of the same time evolutions of the blade deflection previously shown in Fig. 1-4.7. The initial expansion is exponential, which could be observed clearly in the linear shape of the initial enveloping curve in the semi-log plot. The enveloping curves of all the pulses for the same mean wind speed show the same initial exponential coefficient $\lambda_1$, which becomes clear in the collapse of the curves for all the pulses shown in the lower panel of Fig. 2-4.7.

Fig. 2-4.7: Upper panel: semi-log plots of the same time evolutions of the blade deflection for a wind speed regime of 18 m/s. Lower panel: the same curves, but delayed in time for the exponential expansion to build up the same amount of energy contained in their respective pulses.
As it was mentioned before, all the curves could be collapsed into one by shifting them by a certain delay time, \( t_d \), associated with the time required for the exponential expansion to build up the same amount of energy contained in the respective pulse. As it is depicted in Fig. 3-4.7, we took as a time reference the curve for the pulse with the lowest energy (which is the pulse with the longest build-up).

![Fig. 3-4.7: Time delay for each pulse, taking as a time reference the curve for the pulse with the lowest energy.](image)

Then, the value of \( t_d \) for each pulse could be directly computed by using the parameters obtained by fitting each pulse’s exponential curve. Equations (1-4.5) through (5-4.5) show the evaluation of \( t_d \), by equating the exponential expression for each individual pulse with the exponential expression for the reference pulse (that is, collapsing the curves for all the pulses).

\[
U_h = A_0 e^{-\lambda_1 (t_{ref} - t_d)} \tag{1-4.5}
\]

\[
U_{h,ref} = A_0 e^{-\lambda_1 t_{ref}} \tag{2-4.5}
\]

Equating (1-4.5) and (2-4.5)

\[
A_{0,ref} e^{-\lambda_1 t_{ref}} = A_0 e^{-\lambda_1 (t_{ref} - t_d)} \tag{3-4.5}
\]

\[
\frac{A_{0,ref}}{A_0} = \frac{e^{-\lambda_1 t + \lambda_1 t_d}}{e^{-\lambda_1 t}} \tag{4-4.5}
\]

\[
t_d = \frac{1}{\lambda_1} \ln \left( \frac{A_{0,ref}}{A_0} \right) \tag{5-4.5}
\]
Validation for the unstable regime of the hypothesis connecting the time delay and the kinetic energy content of the pulse

In order to validate the hypothesis that the time delay, \( t_d \), is indeed associated with the time required for the exponential expansion to build up the same amount of energy contained in the respective pulse, we analyzed the relationship between the two parameters: pulse kinetic energy, and time delay. In this derivation we also validate the hypothesis that the energy transfer mechanism is exactly the same as in the stable regime, but with a negative value of \( \lambda_1 \), that is, showing amplification instead of attenuation of the oscillatory blade deflection. We shall start by combining equations (1-4.5) and (2-4.5):

\[
U_h = A_0 e^{-\lambda_1 (t_{ref} - t_d)} = A_0 e^{-\lambda_1 t_{ref} e^{\lambda_1 t_d}} = U_{h_{ref}} e^{\lambda_1 t_d}
\]

\[
\frac{U_h}{U_{h_{ref}}} = e^{\lambda_1 t_d}
\]  
(6-4.5)

From the findings about the energy transfer mechanism in Section 4.6, we have that, when normalized by the kinetic energy of the pulse, the blade displacement curves collapse into a unique signal, which scales as the time evolution of the rotor's elastic vibrational energy. This has the units of a flexibility, or the inverse of a stiffness \( 1/K_{el} \), which depends exclusively on mean value of the wind speed. Hence, for all the pulses occurring at the same wind speed, we have:

\[
\frac{U_h}{PulsEner} = 1/K_{el} = \frac{U_{h_{ref}}}{PulsEner_{ref}}
\]

\[
\frac{PulsEner}{PulsEner_{ref}} = \frac{U_h}{U_{h_{ref}}}
\]  
(7-4.5)

Equating expressions (6-4.5) and (7-4.5) we finally obtain:

\[
\frac{PulsEner}{PulsEner_{ref}} = e^{\lambda_1 t_d} \quad \rightarrow \quad \ln\left(\frac{PulsEner}{PulsEner_{ref}}\right) = \lambda_1 t_d
\]  
(8-4.5)

According to equation (8-4.5), if the mechanism of energy transfer found in Section 4.6 for the stable regime also holds for the unstable regime, the logarithm of the ratio between the kinetic energy of any pulse versus the energy of the reference pulse occurring at the same mean wind speed must show a linear relationship with the corresponding time delay for that same pulse, and the slope of that linear relation must be the value of \( \lambda_1 \) at that wind speed.
Fig. 4-4.7 shows a plot of the logarithm of the kinetic energy ratio in equation (8-4.5) versus time delay for wind speeds covering the entire unstable regime. It could be seen that the values for all the pulses show a very clear alignment with a linear relation, and a least squares fitting gives a value of $\lambda_1$ for each wind speed which is in very close agreement with the values obtained by the fitting of $\lambda_1$ in the exponential expansion stage of $U_{hx}$ from the semi-log plots. Fig. 5-4.7 shows a plot of $\lambda_1$ obtained by these two methods for the complete range of wind speeds in the unstable oscillatory regime.

Fig. 4-4.7: Logarithm of the kinetic energy ratio for different pulses occurring at the same mean wind speed versus pulse time delay, for wind speeds covering the entire unstable regime.

The coincidence of the value of $\lambda_1$ obtained from these two different methods, one measuring the expansion as it progresses in time, the other evaluating the time that would have taken for the same expansion to build the same energy delivered by the pulse, proves that the time delay, $t_d$, is indeed associated with the time required for the exponential expansion to build up the same amount of energy contained in the respective pulse. This also validates the hypothesis that the energy transfer mechanism is exactly the same as in the stable regime.
Fig. 5.4.7: Comparative plots of $\lambda_1$ versus mean wind speed for the complete range of wind speeds in the unstable oscillatory regime, obtained by a direct fitting on the exponential expansion stage of $U_{hx}$ from the semi-log plots and form the energy-transfer relation in equation (8-4.5).

5. Funds Usage

The majority of the funds are being used for PhD student support and provision of computational equipment to conduct the numerical simulation efforts which constitute the kernel of the activities of the MEEM Wind-Energy Research Group.

6. Concluding Remarks

The generous support provided by the Donors, Richard and Elizabeth Henes, has proved invaluable to complement other sources of funding in sustaining the research efforts of the Wind-Energy Research Group at MEEM, with outcomes that had been published in leading journals in the field, presented at conferences and seminars, and form the main component of several PhD thesis dissertations.
7. References


