

Sample questions

- Suppose that humans can have one of three bloodtypes: A, B, O. Assume that 40% of the population has Type A, 50% has type B, and 10% has Type O. If a person has type A, the probability that they have hemophilia is 0.05. People with Type B have a 0.03 probability of having hemophilia, and people with Type O have a 0.10 chance of carrying hemophilia. If a person is selected at random from this population:
 - What is the probability that the person has hemophilia?
 - If the person has hemophilia, what is the probability that they have Type A blood?
- A bin contains 8 light bulbs, 6 of which work properly, but the other 2 are defective. You select for purchase 3 bulbs at random and without replacement from the bin.
 - What is the probability that you get two defective bulbs?
 - What is the probability that you get at least one defective bulb?
- The number of pulses a communications channel receives per second can be modeled with a Poisson distribution with a rate parameter of 2.0.
 - What is the probability the channel receives at least 2 pulses in a one second interval?
 - What is the probability that no pulses are received in a 5 second interval?
- Let A and B denote two events in a sample space. You are given that $P(A)=0.3$, $P(A \cup B)=0.8$ and that A and B are independent events. Determine $P(B)$.
- There are 5 people in a hospital suffering from a particular disease. The drug they take has a 20% success rate; that is, each patient has a 0.20 probability of recovery. Assuming that the patients' fates are independent of one another:
 - What is the probability that 4 or more patients recover?
 - What is the probability that at least one patient recovers?
- A standard 52 card deck contains 13 cards (ace,2,3,4,5,6,7,8,9,10, jack, queen and king) from each of 4 suits (spades, clubs, diamonds, hearts).
 - If you are dealt 4 cards without replacement, what is the probability of getting 2 spades, a heart, and a club?
 - If you repeat the experiment 100 times, each time returning the 4 cards to the deck, what is the probability of getting 4 kings at least one time in the 100 hands?
- In a previous version of the state lottery, the person playing the lottery selected without replacement 6 numbers from a set of 45 numbers $\{0, 1, 2, \dots, 44\}$. At some time later a computer randomly selected without replacement 6 numbers from this same set and you won the big prize if you matched all 6 numbers the computer selected. If your strategy was to buy a ticket for each possible outcome, how many tickets would you have to purchase?
- Let X denote a continuous random variable with probability density function $f(x) = 4x^3/15$ for $1 \leq x \leq 2$.
 - Determine the probability that $X > 1.5$.

- (b) Determine the cumulative distribution function $F(x)$ and state the values of $F(x)$ at $x = 0.5$, 1.5 , and 2.5 .
- (c) Find the density function of $Y = X^2$. Be sure to state the domain of $f(y)$.
9. The amount of cereal in a box is not constant, but the distribution can be modeled with a Gaussian distribution with a mean of 16.5 ounces. If the manufacturer is required to fill 90% of the containers with 16 (or more) ounces of cereal, what is the largest allowable value of the standard deviation, s ?
10. The number of pizza orders received at a pizza place follows a Poisson model with a mean rate of 7 per hour.
- (a) What is the probability that the pizza shop goes more than 1/2 hour between orders?
- (b) If it has been 1 hour since the last order, what is the probability that an order arrives in less than 15 minutes?
11. A pizza shop makes deliveries, and the time to make the delivery follows a uniform distribution between 20 and 35 (minutes): $f(x) = 1/15$ for $20 < x < 35$.
- (a) Find the average delivery time and the standard deviation of the delivery times.
- (b) According to Chebyshev's theorem, at least 75% of the delivery times must be between what two values?
- (c) On each trip, the supervisor of the drivers gives a bonus of \$0.10 for each minute below 35. For example, if a driver takes 28 minutes, that is a \$0.70 bonus. What is the average bonus per trip?
12. The lifetime of a lightbulb follows a Gaussian model with a mean of 1000 hours and a standard deviation of 100 hours.
- (a) If I purchase 1 of these bulbs, what is the probability the bulb lasts at least 1100 hours?
- (b) If I purchase 4 of these bulbs, what is the probability at all four of them lasts more than 1200 hours?
- (c) I purchased a bulb that has been operating for 900 hours. What is the probability it lasts another 300 hours?
13. The continuous random variable X has probability density function $f(x) = 1$ for $0 < x < 1.0$.
- (a) Determine the probability density function $g(y)$ of $Y = -\ln(1 - X)$. Be sure to state the domain of $g(y)$.
- (b) Determine the expected value of the function e^X . (This is e , the base of the natural logarithms, raised to the X power).
14. Suppose X and Y are independent random variables, each following a Gaussian distribution. The parameters are:
- $$\mu_x = 20, \mu_y = 25, \sigma_x^2 = 9, \sigma_y^2 = 16$$
- (a) Compute $P(X < 23, Y > 21)$.
- (b) Compute $P(X + Y > 48)$.
- (c) Given that X and Y are independent, what is the expected value of the quantity (XY) , $E(XY)$?

15. The joint density function of (X, Y) is $f(x, y) = x/2$ for $0 < y < x < 2$.
- Compute $P(X < 1)$.
 - Compute $P(Y < 1 \mid X = 1.5)$.
 - Compute $P(X < 1.0, Y < 0.5)$.
16. The joint density function of (X, Y) is $f(x, y) = x/2$ for $0 < y < x < 2$. You are given that $E(Y) = 1/2$, $\text{Var}(Y) = 7/36$, and that $E(X) = 1.0$. Find the correlation between X and Y .
17. You generate 100 independent observations from the pdf $f(x) = 1/4$ for $0 < X < 4$. What is the (approximate) probability that the sum of these 100 observations is less than 190?
18. The time to failure of a device follows the probability model $f(x) = 10/x^2$ for $x > 10$ (X is measured in months).
19. You have 3 such devices operating independently. What is the probability that all 3 survive at least 20 months?
20. You have two such devices operating independently. Let X and Y denote the times to failure of the two devices. What is $P(X > 30 \mid Y > 20)$?
21. The fracture strength of tempered glass average 14 (measured in thousands of pounds per square inch) and has standard deviation 2.
- What is the probability that the average fracture strength of 100 randomly selected pieces of this glass exceeds 14.5?
 - Find an interval that includes, with probability 0.95, the average fracture strength of 100 randomly selected pieces of this glass.
22. Let Y_1, Y_2, \dots, Y_n be independent random variables, each with a beta distribution, with $\alpha = \beta = 2$. Find
- the probability distribution function of $Y_{(n)} = \max(Y_1, Y_2, \dots, Y_n)$.
 - the density function of $Y_{(n)}$.
 - $E(Y_{(n)})$ for $n = 2$.
23. Let Y_1 and Y_2 be independent and uniformly distributed over the interval $(0, 1)$. Find the probability density function of each of the following:
- $U_1 = Y_1 / Y_2$.
 - $U_2 = 2Y_1^2 + 3$.
24. Let Y_1, Y_2, \dots, Y_6 be independent random variables from a standard normal population and let $\bar{Y} = (1/5) \sum_{i=1}^5 Y_i$. What is the distribution of

(a) $W = \sum_{i=1}^5 Y_i^2$? Why?

(b) $U = \sum_{i=1}^5 (Y_i - \bar{Y})^2$? Why?

(c) $\sqrt{5}Y_6 / \sqrt{W}$? Why?

(d) $2(5\bar{Y}^2 + Y_6^2) / U$? Why?

25. Let Y_1, Y_2, \dots, Y_n be independent Poisson random variables with means $\lambda_1, \lambda_2, \dots, \lambda_n$, respectively. Find the

(a) probability function of $\sum_{i=1}^n Y_i$.

(b) conditional probability function of Y_1 , given that $\sum_{i=1}^n Y_i = m$.

26. Resistors to be used in a circuit have average resistance 200 ohms and standard deviation 10 ohms. Suppose 25 of these resistors are randomly selected to be used in a circuit.

(a) What is the approximate probability that the average resistance for the 25 resistors is between 199 and 202 ohms?

(b) Find the approximate probability that the total resistance does not exceed 5100 ohms.

27. To estimate the proportion of unemployed workers in Panama, an economist selected at random 400 persons from the working class. Of these, 25 were unemployed.

(a) Estimate the true proportion of unemployed workers.

(b) Find a 95% confidence interval for the proportion of unemployed workers in Panama.

(c) How many persons must be sampled to reduce the bound on the error of estimation to 0.02 with 95% confidence?

28. Two methods for teaching reading were applied to two randomly selected groups of elementary schoolchildren and were compared on the basis of a reading comprehension test given at the end of the learning period. The sample means and variances computed from the test scores are shown in the accompanying table. Find a 95% confidence interval for $(\mu_1 - \mu_2)$. What assumptions are necessary?

Statistic	Method 1	Method 2
Number of children in group	11	14
\bar{y}	64	69
s^2	52	71

29. If X_1, X_2, \dots, X_n constitute a random sample from the population given by

$$f(x) = \begin{cases} e^{-(x-\theta)} & \text{for } x > \theta \\ 0 & \text{elsewhere} \end{cases}$$

- Show that $\hat{\theta}_1 = \bar{X}$ is a biased estimator of θ .
- What is the bias $B(\hat{\theta}_1)$?
- Find the $MSE(\hat{\theta}_1)$.
- Base on $\hat{\theta}_1$ in a), find an unbiased estimator $\hat{\theta}_2$ of θ .
- What is the $MSE(\hat{\theta}_2)$?

30. If Y_1, Y_2, \dots, Y_n constitute a random sample of size n from an exponential population with parameter θ .

- Find the minimal sufficient statistic of the parameter θ .
- Find the MVUE of the parameter θ .
- Show that the MVUE obtained in b) is a consistent estimator of the parameter θ .
- Find the maximum likelihood estimator of the parameter θ .

31. Suppose that Y_1, Y_2, \dots, Y_n constitute a random sample from the density function

$$f(y|\theta) = \begin{cases} e^{-(y-\theta)}, & y > \theta, \\ 0, & \text{elsewhere} \end{cases}$$

where θ is an unknown, positive constant.

- Find an estimator $\hat{\theta}_1$ for θ by the method of moments.
- Find an estimator $\hat{\theta}_2$ for θ by the method of maximum likelihood.
- Adjust $\hat{\theta}_1$ and $\hat{\theta}_2$ so that they are unbiased. Find the efficiency of the adjusted $\hat{\theta}_1$ relative to the adjusted $\hat{\theta}_2$.

32. Do you believe that an exceptionally high percentage of the executives of large corporations are right-handed? Although 85% of the general public is right-handed, a survey of 300 chief executive officers of large corporations found that 96% were right-handed. Is this difference in percentages statistically significant?

- State the null and alternative hypotheses.
- Calculate the observed value of the appropriate test statistic.
- What is the appropriate rejection region for an $\alpha = 0.01$ level test?
- What is your conclusion?

34. Two sets of elementary schoolchildren were taught to read by using different methods, 50 by each method. At the conclusion of the instructional period, a reading test yielded the results $\bar{y}_1 = 74, \bar{y}_2 = 71, s_1 = 9, s_2 = 10$. We want to see whether evidence indicates a difference between the two population means.

- (a) State the null and alternative hypotheses.
- (b) Calculate the observed value of the appropriate test statistic.
- (c) What is the appropriate rejection region for an $\alpha = 0.01$ level test?
- (d) What is your conclusion?