

## Sample questions

1. How many integral solutions of

$$x_1 + x_2 + x_3 + x_4 = 30$$

Satisfy  $x_1 \geq 2$ ,  $x_2 \geq 0$ ,  $x_3 \geq -5$ , and  $x_4 \geq 8$ .

2. How many ways are there to colour the vertices of the 5-cycle so that adjacent vertices receive different colours?
3. Let  $A(X) = \sum_{n=0}^{\infty} a_n X^n$ , where  $a_n = 3a_{n-1} - 2a_{n-2} + 2$ , and  $a_0 = a_1 = 1$ . Write  $A(X)$  as the quotient of two polynomials.
4. Show that every automorphism of a tree must fix a vertex or an edge.
5. Show that there are at most 5 connected simple planar graphs in which every face has the same degree  $\phi$  and every vertex has the same degree  $\delta > 2$ .
6. For a given graph  $G$  show that the chromatic number  $\chi(G)$  is less than or equal to the maximum degree  $\Delta(G)$ .
7. Construct a cyclic Steiner Triple system of order 13.
8. Construct 2 idempotent mutually orthogonal Latin squares of order 4.
9. Show that a transversal design on  $nk$  points with  $k$  groups is equivalent to  $k-2$  mutually orthogonal Latin squares of order  $n$ .