Sample questions

1. Let G be a finite group containing a subgroup H of index p, where p is the smallest prime divisor of $|G|$. Prove that H is a normal subgroup of G.
2. Prove that every group of order 15 is cyclic.
3. Prove that there is no simple group of order 36.
4. Prove that Euclidean domains are principal.
5. Prove that the maximal ideals of the polynomial ring $\mathbf{C}[x]$ are in bijective correspondence with complex numbers.
6. Prove or Disprove that $X^4 + X^2 + 1 \in \mathbf{Z}_5[X]$ is irreducible.
7. $\mathbf{F}$ be a field of order p^n, p a prime. Show that the automorphisms of $\mathbf{F}$ form a cyclic group of order n.
8. Determine the irreducible polynomial for $\alpha = \sqrt{3} + \sqrt{5}$ over $\mathbf{Q}$.
9. Construct the finite field $\mathbf{F}$ of order 9, and find a generator for the multiplicative group of nonzero elements of $\mathbf{F}$.