Qualifier Exam: Computation Theory

Computer Science

January 6, 2012

Do exactly 6 of the following 8 problems. You must do problem 8.

1. Let Σ be an alphabet and let w be a non-null string in Σ^*. Define the language L_w recursively, as follows.

   $w \in L_w$, and
   if $s \in L_w$, then $ss^R \in L_w$,

   where s^R denotes the reversal of s.

   Prove that for some choice of w,

   i) L_w is regular, *or*
   ii) L_w is not regular and L_w is context-free, *or*
   iii) L_w is not context-free.

2. A *useless* state in a pushdown automaton is never entered on any input string. Show that the problem of determining whether a pushdown automaton has any useless states is decidable.

3. A *queue automaton* is identical to a pushdown automaton, except that the auxiliary data structure is a queue rather than a stack, that is, symbols added to the queue are removed in first-in, first-out order rather than last-in, first-out order.

   Consider the class of languages recognized by queue automata. How does it compare to other established classes of languages (*e.g.,* regular, context-free, recursive, and recursively enumerable)? Justify your answer.

4. (a) Prove that it is undecidable whether a Turing machine M, given input w, enters all of its states.

   (b) Explain why Rice's Theorem *cannot* be used to prove this result.

5. Prove that a language L is decidable iff L can be enumerated in lexicographic order.

6. Compare the classes of problems $TIME(2^n)$ and $\bigcup_k TIME(2^{n^k})$. Are they equivalent? Justify your answer.

7. Recall that coNP is the set of languages L such that $\overline{L} \in$ NP.
   Prove that if NP $\neq$ coNP, then P $\neq$ NP.

8. A *cubic graph* is an undirected graph in which each vertex has degree three.

   The *CUBIC-VC problem* is the VERTEX-COVER problem restricted to cubic graphs. Show that CUBIC-VC is NP-complete.

   Hint: Give a reduction from VERTEX-COVER: Given a graph (V, E) and a positive integer b, is there a subset C of V of size b such that every edge in E has an endpoint in C?

   Show how to "convert" an arbitrary graph G into a cubic graph, with an appropriate modification to the size bound b. The problematic cases in G are vertices with degree $0, 1, 2$ and > 3.