A route for efficient non-resonance cloaking by using multilayer dielectric coating

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An approach for designing transmission cloaks by using ordinary dielectrics instead of meta- and plasmonic materials is proposed and demonstrated by the development of a multi-layer cloak for hiding cylindrical objects larger than the wavelengths of incident radiation. The parameters of the cloak layers were found by using the Genetic Algorithm-based optimization procedure, which employed the reciprocal of total scattering cross width of the cloaked target, derived from the solution of the Helmholtz equation, as the fitness function. The proposed cloak demonstrated better cloaking efficiency than did a similarly sized metamaterial cloak designed by using the transformation optics relations.

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Development of the transformation optics (TO) approaches and metamaterials have opened up opportunities for designing cloaking shells that could make objects invisible[1-5]. However, TO relations required spatially dispersed singular values of the cloak medium constitutive parameters that presented serious challenges for the cloak implementation. Alternatively, approaches based on scattering cancellation have been proposed to realize cloaking[6-8]. In particular, it was shown that scattering from a dielectric target could be cancelled by scattering from the shell made of a plasmonic material[6]. The above cancellation techniques, however, targeted the dominant scattering mode and, therefore, hiding objects much smaller than incident wavelengths, while for larger objects higher order scattering modes deteriorated the cloaking effect. To enhance the quality of scattering suppression, multi-layer cloaking shells were proposed, which utilized secondary reflections from the boundaries of layers[9-10]. Most of the proposed up to date multi-layer cloaks, however, employ layers of plasmonic or other complex artificial materials that are difficult to implement in practice.

Meanwhile, in older studies of the radar detection problem[11-12], it was revealed that coating of an infinite conducting cylinder by a dielectric layer with properly chosen permittivity and thickness could cause significant decrease in backscattering[11]. Even better results for backscattering were observed at a multilayer dielectric coating[12]. Recently, a two-layer dielectric shell was also found efficient for decreasing the total scattering cross width (TSCW) of a cylindrical metallic object [13], although the diameter of the object was much smaller than the radiation wavelength.

In this work we investigate an attractive opportunity to design a multi-layer cylindrical cloaking shell from ordinary dielectric materials ($\varepsilon_r>1$) for hiding objects with diameters exceeding the wavelengths of incident illumination. In addition, in difference from most of works on cloaking, we targeted an essential decrease of the shell thickness compared to the diameter of the object. Illumination by TM mode was considered (with magnetic field directed along the axis of the cylindrical target). We aimed to minimize the TSCW of the cloaked target and employed the Genetic Algorithm (GA) for optimizing the dielectric profile of the multilayer shell.

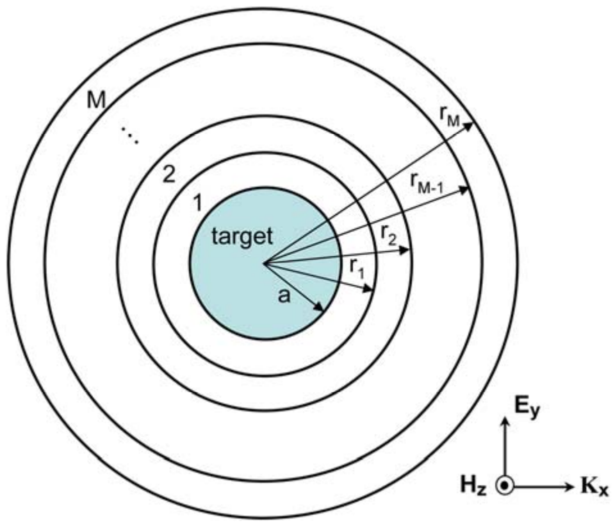

FIG. 1. (Color online) Schematic of the multi-layer cylindrical cloak with the target inside

Scattering from the target covered by a multi-layer shell (layers numbered from 1 to M in Fig.1) was analyzed for the incident wave described by $H_z^i = H_0 \exp(-jk_0 x)$, where H_0 is the magnetic field magnitude and k_0 is the wave number in free space. The radii of the outer boundaries of the layers are denoted by r_m (m=1, 2…M) (Fig.1). The target region and free space are numbered as layer 0 and layer M+1, respectively, for the notation consistence. The material of the target was taken to be a perfect electric conductor (PEC), however, an object made of any material covered by a thin PEC layer could be taken instead.

The magnetic field H_{zm} in the m_{th} cloak layer for TM illumination could be found from the solution of the Helmholtz equation analogous to that used for the electric field at consideration of TE case[9]:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{r}{\varepsilon_{\phi m}}\frac{\partial H_{zm}}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \phi}\left(\frac{1}{\varepsilon_{rm}}\frac{\partial H_{zm}}{\partial \phi}\right)+k_0^2\mu_{zm}H_{zm}=0, \tag{1}$$

The presence of only one z-component of the permeability tensor in Eq.(1) makes the TM case advantageous for implementation since it allows for avoiding magnetic materials in the shell. After separation of variables, the general solution of Eq. (1) can be expressed as:

$$H_{zm}=\sum_{n=-\infty}^{\infty} a_{mn}\left[J_{V_{mn}}(k_m r)+\tilde{R}_{m(m-1)n}H_{V_{mn}}^{(2)}(k_m r)\right]\cdot\exp(jn\phi), \tag{2}$$

where ‘n’ is the order of azimuthal harmonic function, a_{mn} is unknown coefficient,

$k_m = k_0\sqrt{\mu_{zm}\varepsilon_{\phi m}}$ is the wave number in the m_{th} layer, and $V_{mn} = n\sqrt{\varepsilon_{\phi m}/\varepsilon_{\rho m}}$ is the order of the Bessel function. J(x) is the Bessel function of the first kind, while $H^{(2)}(x)$ is the Hankel function of the second kind representing the outward (in the +r direction) traveling cylindrical wave with time dependence exp(jωt), and $\tilde{R}_{m(m-1)n}$ is the total outward scattering coefficient at the interface between m_{th} layer and $(m-1)_{th}$ layer. The total scattering wave at the interface $r = r_{m-1}$ consists of two components: the wave resulting from partial reflection of the inward propagating wave and controlled by the coefficient $R_{m(m-1)}$ and an additional wave coming from the region $r < r_{m-1}$ and governed by the term, which takes into account the wave totally scattered at the interface $r = r_{m-2}$: $T_{(m-1)m}\cdot\tilde{R}_{(m-1)(m-2)}\cdot\left(1-R_{(m-1)m}\cdot\tilde{R}_{(m-1)(m-2)}\right)^{-1}\cdot T_{m(m-1)}$, where $T_{m(m-1)}$ is transmission coefficient of inward propagating wave at the above interface, while $R_{(m-1)m}$ and $T_{m-1)m}$ are direct reflection and transmission coefficients for outward propagating wave at the same interface. Therefore, $\tilde{R}_{m(m-1)n}$ at any interface can be defined by the given below expression (where the index 'n' is omitted for shorter notation)[14]:

$$\tilde{R}_{m(m-1)} = R_{m(m-1)} + T_{(m-1)m}\cdot\tilde{R}_{(m-1)(m-2)}\cdot\left(1-R_{(m-1)m}\cdot\tilde{R}_{(m-1)(m-2)}\right)^{-1}\cdot T_{m(m-1)}\ , \tag{3}$$

The direct reflection and transmission coefficients have been derived by matching the tangential E-field and H-field components at the interfaces:

$$\begin{cases} R_{m(m-1)} = \dfrac{j'j_{-1} - \eta_{m-1}/\eta_m j'_{-1} j}{\eta_{m-1}/\eta_m j'_{-1}h - j_{-1}h'} \\ T_{m(m-1)} = \dfrac{2i}{\pi k_m r_{m-1}}\cdot\dfrac{1}{\eta_{m-1}/\eta_m j'_{-1}h - j_{-1}h'} \end{cases} \quad \begin{cases} R_{(m-1)m} = \dfrac{h'h_{-1} - \eta_{m-1}/\eta_m h'_{-1}h}{\eta_{m-1}/\eta_m j'_{-1}h - j_{-1}h'} \\ T_{(m-1)m} = \dfrac{2i}{\pi k_{m-1} r_{m-1}}\cdot\dfrac{1}{j'_{-1}h - \eta_m/\eta_{m-1} j_{-1}h'} \end{cases} \tag{4}$$

where, $j = J_{V_{nm}}(k_m r_{m-1})$, $j' = J'_{V_{nm}}(k_m r_{m-1})$, $j_{-1} = J_{V_{n(m-1)}}(k_{m-1}r_{m-1})$, $j'_{-1} = J'_{V_{n(m-1)}}(k_{m-1}r_{m-1})$, $h = H^{(2)}_{V_{nm}}(k_m r_{m-1})$, $h' = H^{(2)\prime}_{V_{nm}}(k_m r_{m-1})$, $h_{-1} = H^{(2)}_{V_{n(m-1)}}(k_{m-1}r_{m-1})$, $h'_{-1} = H^{(2)}_{V_{n(m-1)}}{}'(k_{m-1}r_{m-1})$, and η_m is the wave impedance in the m_{th} layer.

With account for Eq.(4), the Eq.(3) could be employed for deriving recursively the total scattering coefficient for the outer surface of the shell, however, initialization of the recursive procedure demanded knowing of the scattering coefficient R_{10} at the interface between the PEC target and the first layer of the shell. For the case of TM wave incidence this coefficient could be defined by the equation[15]:

$$R_{10n} = -J_n^{'}(k_1 a) / H_n^{(2)'}(k_1 a)\,, \tag{5}$$

After the total scattering coefficient at the outer cloak surface for each order of cylindrical wave $\tilde{R}_{(M+1)Mn}$ is found, the scattered field of the multilayer cloak could be determined by using the expression $H_z^s = H_0 \sum_{n=-\infty}^{\infty} j^{-n} \tilde{R}_{(M+1)Mn} H_n^{(2)}(k_0 r) e^{jn\phi}$. In the far field approximation we employed the asymptotic formula of Hankel function, therefore, the TSCW of the cloak σ could be derived as:

$$\sigma = \int_0^{2\pi} \frac{\left|H_z^s\right|^2}{\left|H_0\right|^2} r \cdot d\phi = \frac{2}{\pi k_0} \int_0^{2\pi} \left| \sum_{n=-\infty}^{\infty} (-1)^{-n} \tilde{R}_{(M+1)Mn} e^{jn\phi} \right|^2 d\phi \tag{6}$$

The obtained expression provided an opportunity for quantitative characterization of the cloak performance and was further used to define the fitness function (1/σ) in the cloak design optimization procedure based on GA. The goal of this procedure was to search for the relative permittivity values exceeding or equal to 1 for each dielectric layer, which would provide the minimal possible value of σ. The efficient elitist strategy with tournament selection[16] has been employed for optimization. Taking into account the trade-off between implementation simplicity and optimization diversity, an eight-layer cloak of equi-thick layers has been explored here. At the given value of target radius 'a' equal to either 0.75λ, or 1.0λ the thickness of each of 8 layers was controlled by the optimal value of 'b'.

When applying the GA, the shell parameters {ε_i, b} were coded by binary digits and the search domain for 'ε_i' was chosen between 1 and 128(2^7). The search domain of the outer radius 'b' was defined by the range between 1.05a and 2a that provided for diverse options for the layer thicknesses. To produce next generations of the cloak parameters, the probabilities of crossover and mutation were set to be 0.8 and 0.1, respectively[16]. The results of the cloak parameter optimization for two chosen targets with a=0.75λ and a=1.0λ are presented in Table I and in Fig. 2.

TABLE I. Optimized dielectric constant values of the layers and the outer radius of the cloak

a/λ	ε_1	ε_2	ε_3	ε_4	ε_5	ε_6	ε_7	ε_8	b/λ
0.75	1.0	65.0	65.0	17.0	3.0	3.0	4.0	58.0	0.8695
1.0	1.0	1.0	65.0	65.0	25.0	6.0	50.0	65.0	1.1125

As seen from the table and Fig. 2, radial dispersion of the cloak permittivity in the optimized shell cannot be described by a simple monotonic dependence for both target sizes. One of noticeable features is the need to have an air gap between the target and the cloak. It is interesting to note that in the earlier work devoted to the solution of the backscattering problem for conducting cylinders coated by dielectric shells it was also found useful to introduce an air gap with up to the wavelength width[17]. Another specific feature of the optimized dielectric profiles is their U-shape (Fig. 2). In addition, it is worth noting equal permittivity values in some neighboring layers that could further simplify the cloak implementation.

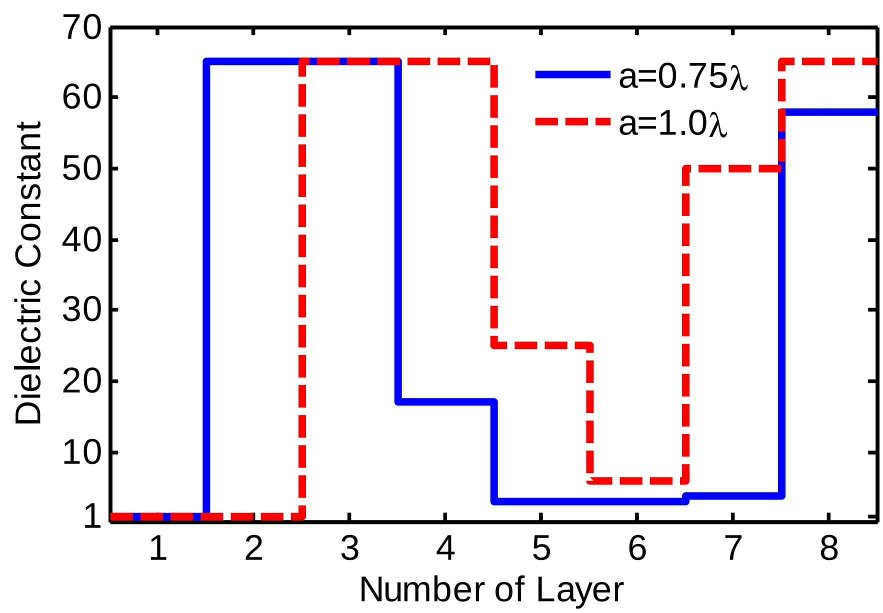

Fig. 2. (Color online)Optimized radial dielectric profiles of the multi-layer shells for two targets with different radii.

An important result of optimization is that the optimized shell thickness is only 16% of the target radius at a=0.75λ and 11.25% at a=1.0λ, respectively. As known, the thickness of cloaks designed by using the TO approach was usually comparable to ‘a’, i.e. was 8-9 times bigger than that of the proposed cloak. Among other designs only the mantle cloak had comparable shell thickness[8].

The presented in Table 1 optimized values of shell parameters were used for calculating the TSCW for cloaked and bare targets by using Eq.6. For the target with the radius of 0.75 λ the TSCW was found to be only 1.24λ, i.e. 50% of the TSCW value for the bare target (2.48λ), while for the target with the radius of 1.0 λ the TSCW of the cloaked target (2.15λ) was found to be 62% of the TSCW value for the bare target (3.47λ). It should be mentioned that earlier works on backscattering from PEC targets coated by multi-layer shells also did not provide perfect scattering cancellation and mentioned some restrictions in scattering reduction[11-12].

To additionally ensure advantages of the optimized multi-layer dielectric cloaks beyond their material simplicity, the performance of one of them was compared with that of the cloak having the same dimensions (a=0.75λ, b=0.8695λ) and the same thicknesses of 8 layers (0.015λ) but designed by using the TO approach, in particular, possessing with radial dispersion of the effective permittivity and values of other parameters satisfying the given below relations derived for TM wave incedence[5]:

$$\mu_z = 1, \varepsilon_\theta = [b/(b-a)]^2, \varepsilon_r = [b/(b-a)]^2[(r-a)/r]^2 \tag{7}$$

Although a TO approach-based cloak with such thin layers as those obtained at optimization of the multi-layer dielectric cloak (0.015λ) could not be implemented using conventional metamaterials composed of resonance elements with dimensions of about 0.1λ, there is no fundamental restriction for the theoretical comparison of two cloaks. Providing for the prescribed by Eq. (7) parameters of the layers μ_z=1, ε_θ=52.92 and ε_{ri}<1 (i=1…8) would demand in practice meta- or plasmonic materials. Since the above mediums are the resonance ones, their operation is frequency dependent. However, for the desired comparison of two cloaks these specifics could be ignored.

The multi-layer dielectric cloak could perform at any frequency if the dielectric profile presented in Table 1 is provided. However, since geometric parameters of this cloak have been normalized with respect to the wavelength of incident radiation, they should be varied accordingly for various operation frequencies.

Taking into account the above considerations, the two cloak performances could be compared at arbitrary frequency f_{work}, while the dimensions of the multi-layer dielectric cloak should be determined by using λ_{work}= c/ f_{work}. The presented below results have been obtained for f_{work}=8 GHz, i.e. for the cloaks with the dimensions defined by λ_{work}=3.75cm. The commercial software COMSOL Multiphysics 4.2 was used to simulate wave incidence on multi-layer dielectric and TO-based cloaks. Snap-shots of H_z field distributions of two cloaks with the same target are compared in Fig. 3. As seen from Fig. 3a, strong distortions of the wave front and a "shadow' behind the target are clearly observed in the latter case. In contrast, both cloaks provide decreased reflections and essentially less expressed shadows, however, the multi-layer dielectric cloak performance is obviously better.

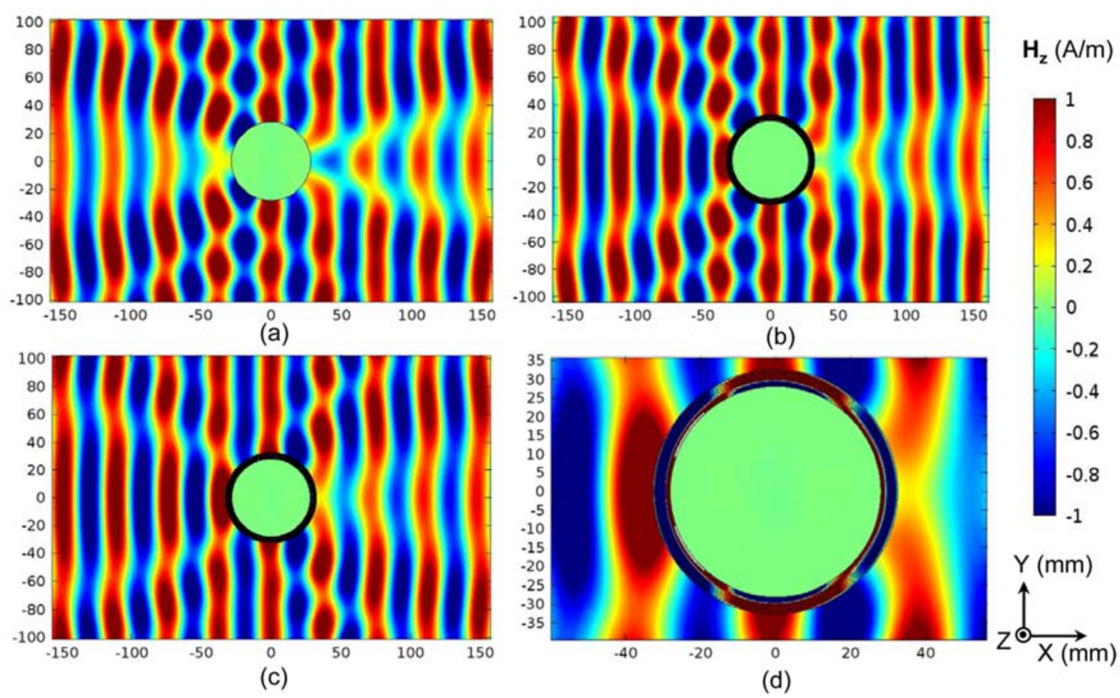

FIG. 3. (Color online) Snap-shots of H_z distributions for f_{work}=8 GHz at TM wave incidence on: (a) bare metallic cylinder with the radius 0.75λ_{work}; (b) the same target cloaked by the shell with material parameters prescribed by Eq. (7); (c) the same target cloaked by the multi-layer dielectric shell; (d) zoomed-in view of (c).

The above observations were also justified by TSCW comparison for two cloaks obtained by using COMSOL-based simulations of integrated far-fields of scattered waves. The obtained TSCW values for the bare target and for the target cloaked by either the TO based cloak or the multi-layer dielectric cloak were 9.56 cm (2.55λ_{work}), 7.60 cm (2.026λ_{work}) and

4.80 cm (1.28λ_{work}), respectively. From the obtained values it is obvious that the TO-based metamaterial cloak essentially concedes to the simple multi-layer dielectric cloak. It is worth mentioning that the presented above TSCW data for the bare and dielectric coated targets are consistent with the respective TSCW values found by using the analytical Eq. 6 (2.48λ and 1.24λ, respectively).

The presented in Fig. 3d zoomed-in image of H-field shows that the performance of the multi-layer dielectric cloak is accompanied by trapping of a part of incident electromagnetic energy inside the shell and transferring it along the shell circumference as a kind of azimuthal surface wave. The observed by-passing of the target could possibly contribute in reducing the TSCW.

Integration of the simulated far-fields was also employed for obtaining the TSCW dependencies on frequency for targets with the radii of 0.75λ_{work} and 1.0λ_{work}. The obtained TSCW spectra for frequency swept from 6 GHz to 10 GHz for the bare and cloaked targets are presented in Fig. 4. As seen from the figure, the cloaking effect (the drop of the TSCW below the value characteristic for the bare target) is observed near 8.0 GHz in both cases, but for the larger target the TSCW drop does not exceed 38%. The latter result agrees with noticed in other works trend toward decreasing the cloaking effect for larger targets[6] due to enhancement of the higher order scattering modes. However, in the presented cloak designed to provide maximal suppression of total scattering from the cloaked target, the changes in contributions from various scattering modes in responses from larger targets should produce smaller effect on cloaking than that in cloaks aimed on suppression of only the dominant mode. It is also worth noting that the cloaking effect bandwidth in Fig. 4 is relatively wide (more than 5%) for both targets. Since no optimization of this parameter was performed in the proposed design, there is a potential for increasing it further in future work.

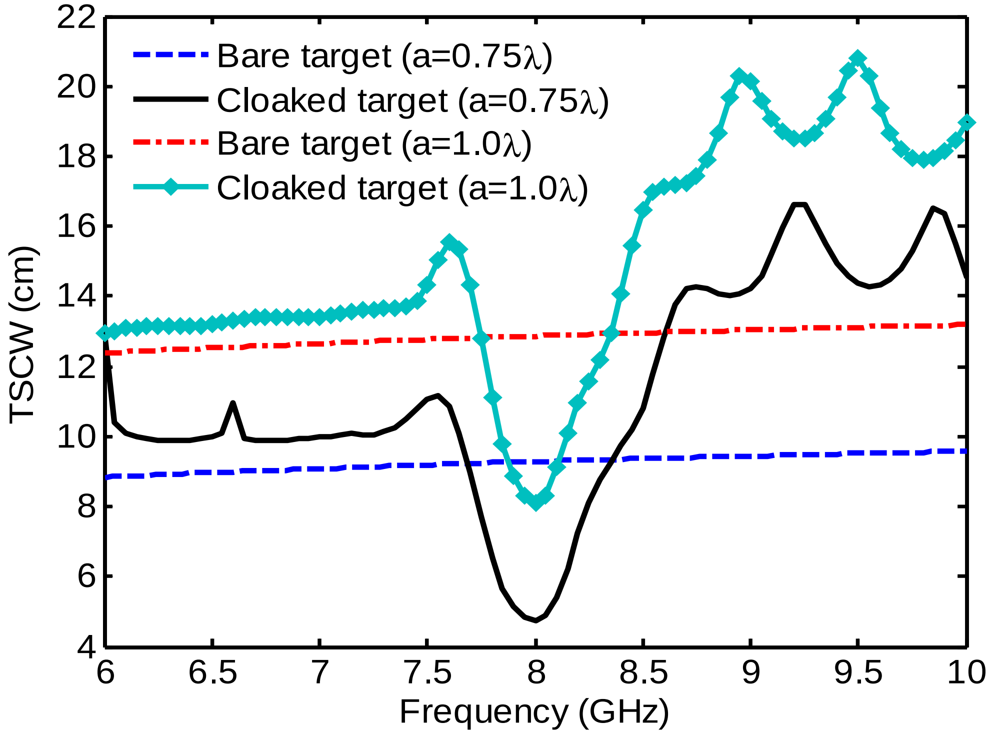

FIG. 4. (Color online) The TSCW spectra obtained by integrating the simulated far-fields for the cloaked and bare targets of two different sizes. Dimensional parameters of the multi-layer dielectric cloaks in both cases were fixed for the wavelength corresponding to f_{work} =8 GHz.

In conclusion, we have developed an approach for designing multi-layer dielectric cloaks that could be easily scaled to various frequency ranges. In particular, it was demonstrated that a 50% TSCW reduction could be achieved for a metal target with the radius of 0.75λ, when it's covered by a shell composed from only few dielectric layers each of 0.015λ thick. It was shown that the multi-layer dielectric cloak essentially outperformed the similarly sized metamaterial cloak designed according to the TO requirements. The design procedure developed in this work could be used for further advancing the cloak parameters and for adjusting it to practical needs.

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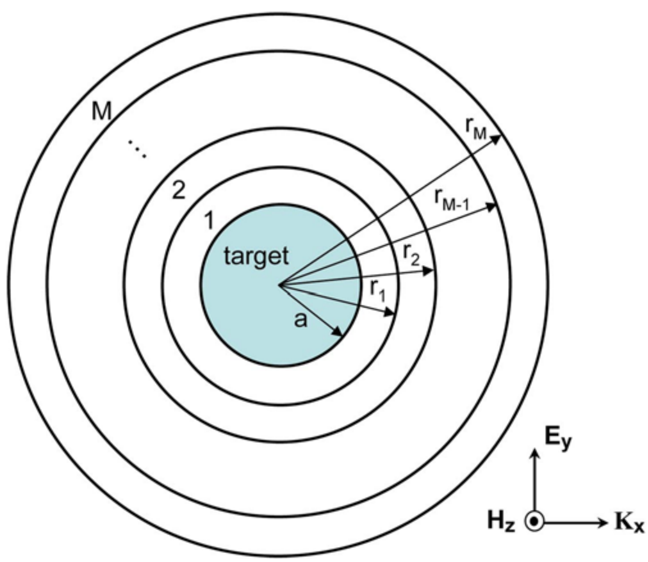
M
2
1
target
a
r_1
r_2
r_{M-1}
r_M
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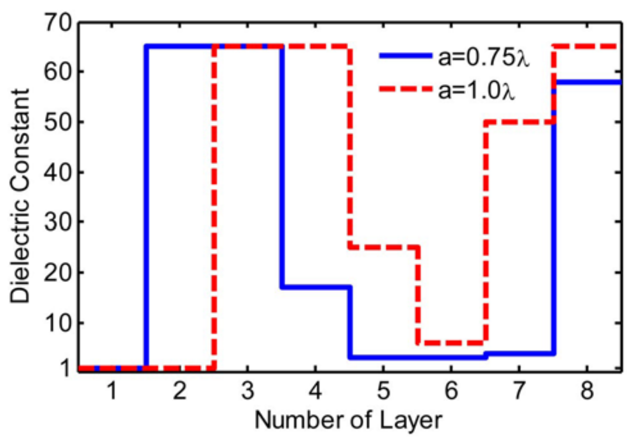
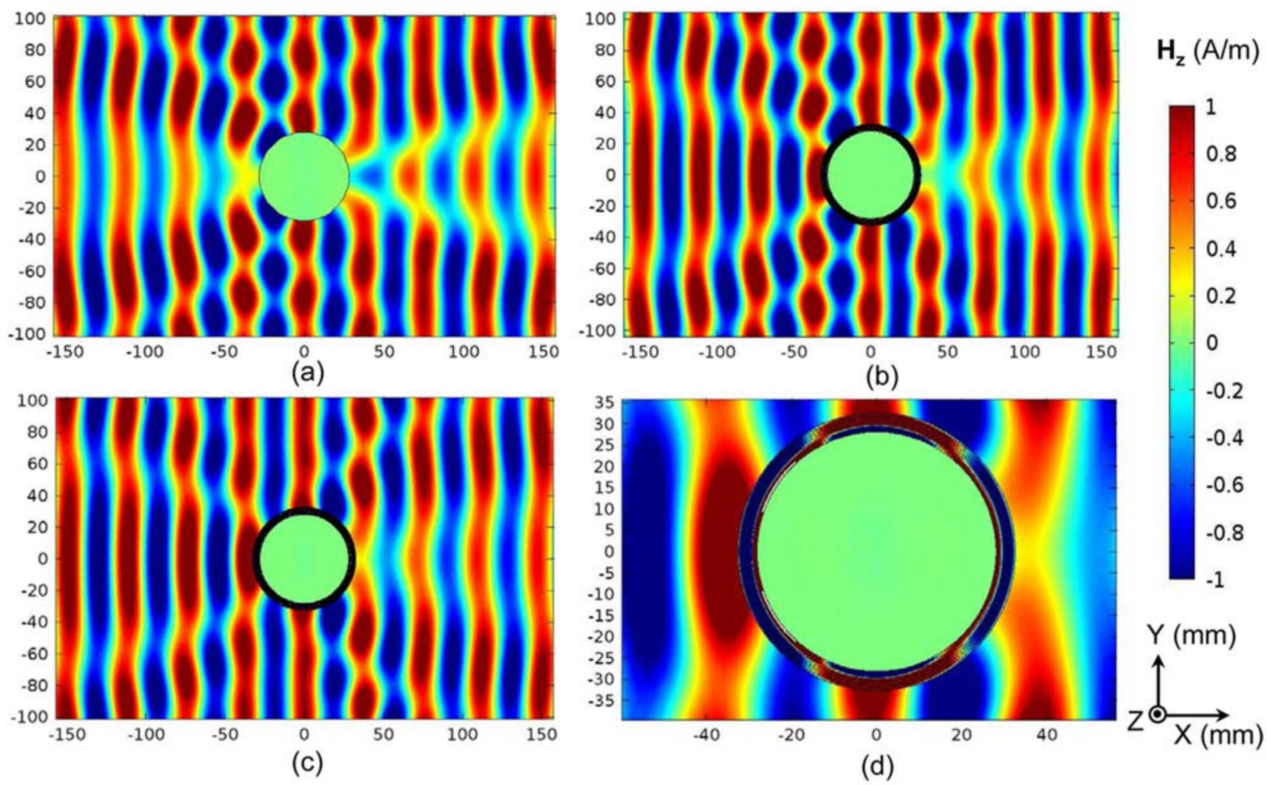
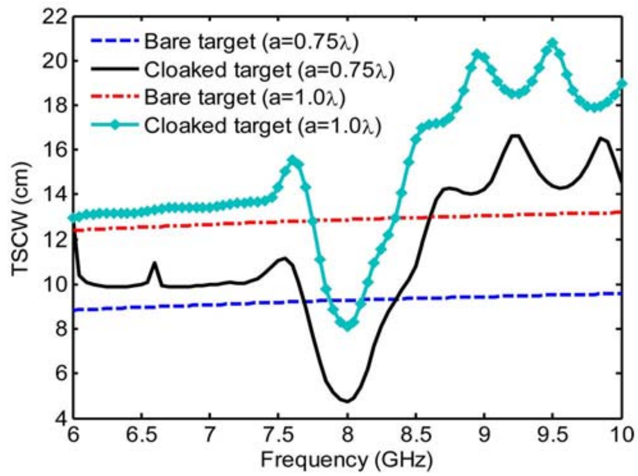


Dielectric Constant
70
60
50
40
30
20
10
1
1
2
3
4
5
6
7
8
Number of Layer
a=0.75λ
a=1.0λ



Hz (A/m)
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
(a)
(b)
(c)
(d)
Y (mm)
Z
X (mm)


Bare target (a=0.75λ)
Cloaked target (a=0.75λ)
Bare target (a=1.0λ)
Cloaked target (a=1.0λ)
TSCW (cm)
Frequency (GHz)
22
20
18
16
14
12
10
8
6
4
6
6.5
7
7.5
8
8.5
9
9.5
10