

# Outline

The Real Analysis Qualifying Exam covers material from a standard two-semester undergraduate course, including the following:

1. Properties of the real numbers
  - (a) Real numbers, rational and irrational numbers
  - (b) The denseness of the rationals
  - (c) The completeness axiom ("least upper bound" property)
2. Sequences
  - (a) Definition of limit (finite and infinite), basic limit theorems
  - (b) Monotone sequences
  - (c) Bolzano-Weierstrass theorem
  - (d) Cauchy sequences and completeness
  - (e) Subsequences, limsup and liminf
  - (f) Series and partial sums, including convergence tests
3. Functions
  - (a) Continuity and uniform continuity
  - (b) Pointwise and uniform convergence
  - (c) Power series and radius of convergence
4. Differentiation
  - (a) Basic rules (chain rule, product rule)
  - (b) Mean Value Theorem, Taylor's Theorem
  - (c) Taylor series and power series
5. Integration
  - (a) Riemann integral
  - (b) Basic theorems (continuous functions are integrable, Integral Mean Value Theorem, etc.)
  - (c) Fundamental Theorem of Calculus
  - (d) Improper integrals

These topics are covered in many standard undergraduate real analysis textbooks, for example:

- Kenneth A. Ross, *Elementary Analysis: The Theory of Calculus*, Springer.
- Jerrold Marsden and Michael Hoffman, *Elementary Classical Analysis*, Freeman Pub. Co.
- William Wade, *An Introduction to Analysis*, Prentice Hall.

In particular, the student should know the statement **and proof** of the following theorems and results:

1. Bounded monotone sequences converge.
2. Convergent sequences are Cauchy sequences.
3. The Bolzano-Weierstrass theorem
4. Absolutely convergent series converge.
5. Continuous functions on a closed interval attain their maximum.
6. The intermediate value theorem
7. The uniform limit of continuous functions is continuous.
8. Rolle's theorem or the Mean Value Theorem
9. If a function is monotone on  $[a,b]$ , then it is Riemann integrable on  $[a,b]$ .
10. The fundamental theorem of integral calculus