Design Theory: Comprehensive Exam

Instructions

- This is a 3-hour examination.
- You are expected to solve at least four problems of the following six problems.
- Please start each problem at the top of a new page.

Problems

1. Give an explicit construction of a 2-(10, 3, 2) design containing two repeated blocks.

2. Let \( \mathcal{D} \) be a collection of 30 subsets (called blocks) of a set \( X \) containing 21 points, with the properties that every point of \( X \) is contained in exactly 10 blocks of \( \mathcal{D} \), and every two points of \( X \) occur together in exactly three blocks. Prove that all blocks are of size 7.

3. Determine if a 2-(\( v \), \( k \), \( \lambda \)) design with the following parameters exists:
   
   (a) \( v = 16 \), \( k = 6 \), \( \lambda = 2 \).

   (b) \( v = 151 \), \( k = 25 \), \( \lambda = 4 \).

   If a design exists, give an example of such design; otherwise, prove that a design with the given parameters does not exist.

4. Describe the construction of a Hadamard matrix of order 44.

5. Give an example of a cyclic \((15, 7, 3)\) difference set.

6. Describe the construction of a resolvable design with 27 points and 27 blocks of size 9 such that:
   
   - Every point is contained in exactly 9 blocks.
   - The points are partitioned into 9 groups of size 3 so that any two points which belong to the same group do not appear together in any block, while any two points which belong to different groups appear together in exactly three blocks.